ICTMT 11
11th International Conference on Technology in Mathematics Teaching

Bari, 9th – 12th July 2013
Department of Mathematics - University of Bari

Conference Proceedings

edited by
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THE RECONSTRUCTION OF MEANING FOR THE CONGRUENCE OF TRIANGLES WITH TURTLE GEOMETRY

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In the present study, ‘Turtleworlds’ a programmable Turtle Geometry medium, is utilized by teachers and students as a means of exploring the congruence of triangles with the help of a half-baked microworld. This activity brought to light a relation which, somehow, brings order to chaos, creating categories (classes) of triangles, each one represented by the unique, constructed triangle. It was also shown that in order to arrive at the process of proof as documentation, it is essential to begin with a generalization of observations, formulation of arguments and their articulation in unified reasoning, so as to enable the student to understand and effectively formulate more formal proofs.

INTRODUCTION

Handling geometrical concepts with the ‘Turtleworlds’ involves an approach which is different from that of traditional teaching. A big difference between the Turtle Geometry and the Cartesian Geometry derives from the concept of intrinsic attributes of geometrical shapes. An intrinsic attribute is one that depends exclusively on the particular shape, not on the relation of the shape to a system of reference (Abelson & diSessa, 1980).

The books of Euclid rely on the three conditions for triangle congruency: the side-angle-side (SAS), the side-side-side (SSS) and the side and two angles (SAA). These three conditions for congruency (SAS, SSS, SAA) are subsequently used in the books of Euclid to prove many more propositions. (Jones & Fujita, 2013). As Freudenthal (1983) argues “this artificial system of congruent triangles has been canonised in the traditional school geometry” (Freudenthal, p. 342, 1983).

In the Euclidean geometry taught in Greek schools, the congruence of triangles is regarded as the possibility of coincidence by superposition, and is proved following a typical scheme of logical reasoning using the three given “criteria of equation” (Argyropoulos et al., 2003).

In ‘Turtleworlds’ the superposition of two different triangles can be replaced by the construction of a triangle, so that, students to be able to understand the equality of triangles as an equivalence class of all triangles of the level. So, “learning is not taken as a simple process of the incorporation of prescribed and given knowledge, but rather as the individual’s (re)construction of geometry” (Laborde et al., p. 278, 2006)

METHODOLOGY OF RESEARCH

We followed the above approach for all three criteria of triangles congruence with students of the first grade of senior high school, who had prior experience in the use of Logo (in Turtleworlds, the elements of a geometrical construction can be expressed in a Logo procedure), and also with teachers of mathematics during a teacher training programme on the use of digital technologies. We focused in the way students and teachers perceived an ‘other’ approach of congruence of triangles which was mediated by ‘Turtleworlds’.

In the classroom:

The students worked on the computer in pairs for two teaching hours. They were asked to handle a half-baked microworld in the Logo environment.
In ‘Turtlewords’, what is manipulated is not the figure itself but the value of a variable of a procedure. This is done by means of a variation tool, which is activated upon clicking on the trace of the figure constructed by a variable procedure. (Kynigos, 2007).

The students’ actions were recorded (by saving files and images). The researcher coordinated the activities and recorded the students’ observations and the groups’ discussions as well.

In the teacher training programme:

The teachers worked on the computer individually. They were asked to handle the same half-baked microworld in the Logo environment as the students did, and their actions were recorded (by saving files and images).

In both cases there followed a discussion, with the researchers acting as coordinators at each stage of the procedure.

PROCEDURE AND FINDINGS

With ‘Turtleworlds’, we aimed at the construction of the triangle using the least possible data. That is, if we construct a triangle given the lengths of two of its sides and the angle contained in these sides, then this triangle will be unique. This observation is related to our initial reference to the intrinsic qualities of the geometrical shape. Initially, in order to construct the triangle, we used all of its main elements: the three sides a, b, c, and two angles, x and y, (obviously, the third angle can be calculated). The half-baked microworld we suggested is shown in Figure 1.

```
for triangle :a :b :c :x :y
  fd :a
  rt 180-:x
  fd :b
  rt 180-:y
  fd :c
end
```

Fig. 1. Half-baked microworld ‘triangle’

In the classroom:

The students activated the variation tool and, correcting the occurring shape, they constructed a triangle; rather, they closed the triangle. In the discussion that followed they realized that the triangle was different for each group. The students observed that, playing with the variation tool, we can construct an infinite number of triangles, each one different from the rest. In this way, our set of reference was defined: it is the set of all the triangles of the plane.

In the teacher training programme:

A question posed by the teachers led the teachers’ group to further investigation, discussion and experimentation with the software, so as to give an answer to the question: “How can we be sure that the triangle has been closed?” The suggestions always led to a dead-end like (Fig. 2).
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Fig. 2. Teacher suggestion 1

Others that use repetition (Figure 3):

```plaintext
for triangle3 :a :b :c :x :y
  fd :a
  rt 180:-x
  fd :b
  rt 180:-y
  fd :c
  rt :x+:y
end
```

triangle3 100 120 150 80 75

Fig. 3. Teacher suggestion 2

The solution was given by the researcher with the use of the commands (see Figure 4).

```plaintext
make  "d distancet oxy
  if :d<0.5
    [setheading 0 0]
```

Fig. 4. Solution

Next, we invited the students to modify the code in order to construct triangles with a specific (fixed) side length. In this way we limited both the use of variation tool and the number of possible triangles at the same time. The code that occurred is shown in Figure 5:

```plaintext
for 3SS :a :b
  fd 100
  rt 180:-a
  fd 150
  rt 180:-b
  fd 120
end
```

Fig. 5. Half-baked microworld ‘SSS’

Each student or group of students constructed their own triangle, which will also be unique in the sense that any change in the angles a and b will result in the construction of the same triangle.
The measure of the two angles (53 and 41) that makes the triangle close became a new topic of discussion among the teachers, when the results differed from these measures even slightly.

In the next activity we once again asked the students to correct and to “rediscover” the initial code by giving fixed values on one side and the adjoining angles, and to “rediscover” the uniqueness of the triangle they can construct. The result is shown in Figure 6.

```
for ASA : a : c
fd : a
rt 180-40
fd 150
rt 180-60
fd : c
end
```

**Fig. 6. Half-baked microworld ‘ASA’**

We discussed with the students the meaning of their previous observations. The observation that in the two previous constructions the occurring triangles were unique led them to fully understand the existence of triangles congruence: when we compare two triangles, it is always the same triangle that appears at two different instances. Following the above observations, we invited the students to experiment by developing the code that corresponds with the criterion SAS (SAS congruency theorem) (side-angle-side) and by playing with the triangle.

The question we posed next refers to the case when the equal angle is not the one contained in the two equal sides (SSA side-side-angle) and we invited the students to formulate a code which materializes the above condition, to apply it and to investigate the multitude and the kind of triangles that can be constructed playing with the the variation tool. During the discussion prior to the investigation with ‘Turtleworlds’, the students excluded the possibility of triangles congruence “since none of our familiar criteria is present.” During the discussion about the results, there occurred observations like: there are two triangles we can construct or the triangle is unique. These observations inspired a heated discussion that resulted in searching for a fourth criterion of equality of triangles, whose formulation and validity had to be explored. Zodik & Zaslavsky (2007) report a discussion among their students about the same topic:

Teacher: Are they congruent according to SAS?

Student: In both of them there isn’t SAS.

Teacher: But are they congruent or not?

Student: Yes they are. (Zodik & Zaslavsky, p. 2031, 2007)

In order to investigate the condition, they developed the code shown in Figure 7. With the appropriate use of the variation tool, this code gave us two different triangles, thus two different categories (classes) of triangles (Fig. 7 and Fig. 8).

An issue of concern that was brought about by this activity is the issue regarding the use of rules and the primitive epistemology underlining or even promoted sometimes in the teaching of mathematics, that is, anything that does not satisfy the rule is wrong: The case of the criterion Side-Side-Angle, when the equal angle is not the contained one, does not exclude the congruence of the triangles.
CONCLUSIONS

Each individual must reconstruct knowledge. Of course, one does not necessarily do this alone. Everyone needs the help of other people and the support of a material environment, of a culture and society (Papert, 1990).

The reconstruction of knowledge mediated by the microworlds gives new meaning to the three criteria-congruency theorems.

The initial construction of triangles using their basic elements (three sides and three angles) freely changing introduces us to the concept of set of reference, that is, the set of all the triangles of the plane. The construction of triangles with less than the six basic elements, in some way, brings order to chaos, creating categories (classes) of triangles, each one represented by the unique, constructed triangle. ‘Turtleworlds’ a programmable Turtle Geometry medium, provides us with the means to investigate through modification, while it restricts us in a shape that contains its qualities and ultimately represents an infinite multitude of shapes.

Furthermore, the concept of mediation is appeared in this intervention. The initial use of ‘Turtleworlds’ transformed to a mediating role for understanding a mathematical content (Drijvers et al., 2010). The mediating role of the tool gave the opportunity to both teachers and students to develop their own mathematical meanings. The transition from the personal perspective, to the developing of a framework which consciously approaching mathematical concepts, is a process that is neither spontaneous nor provided. The existence of a teaching cycle, which is constructed so as to include semiotic activities and a collective mathematical discourse, with the role of teacher-trainer to focus on facilitating and guiding the evolution of signs are particularly important (Bartolini Bussi & Mariotti, 2008).
The teachers in training, resisted in a way this approach, and were very skeptical about the closing of the triangle. The cause of this resistance is the demand (or necessity?) for a fixed procedure leading to the proof, which is usually applied at school. Proof is not only the typical one that the student will encounter in senior high school. The concept of proof as documentation begins with the generalization of observations, the formulation of arguments, and their articulation in unified reasoning. It is necessary for the students to proceed through all these stages, before they can start comprehending and become able to formulate more typical proofs. Otherwise, they will regard mathematical proofs as a meaningless ritual. Indicative of this is the behaviour of students who, not recognizing the characteristics of SAS, but encountering an ASS case, refuse to even speculate on the possibility of congruent triangles before they see them taking shape with the navigation of the turtle.

REFERENCES


